7.8 Videos Guide

7.8a

- Types of improper integrals
  - Infinite interval: $\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_{t}^{a} f(x)dx$ or $\int_{a}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{a}^{t} f(x)dx$
  - Infinite discontinuity: $\int_{a}^{b} f(x)\,dx$, where there is $c \in [a, b]$ such that $\lim_{x\to c} f(x) = \pm\infty$. In this case we have
  $$\int_{a}^{b} f(x)\,dx = \lim_{t\to c^-} \int_{a}^{t} f(x)dx + \lim_{t\to c^+} \int_{t}^{b} f(x)dx$$

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

- $\int_{1}^{\infty} \frac{e^{-1/x}}{x^2}\,dx$
- $\int_{0}^{\infty} \sin\theta\, e^{\cos\theta}\,d\theta$

7.8b

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

- $\int_{2}^{\infty} \frac{dv}{v^2+2v-3}$
- $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$

7.8c

Exercise:

- For what values of p is the integral $\int_{1}^{\infty} \frac{1}{x^p}\,dx$ convergent?

7.8d

Theorem (statement):

- Comparison Theorem: If $f(x) \geq g(x) \geq 0$ (both continuous) for $x \geq a$, then
  a) If $\int_{a}^{\infty} f(x)\,dx$ is convergent, then so is $\int_{a}^{\infty} g(x)\,dx$
  b) If $\int_{a}^{\infty} g(x)\,dx$ is divergent, then so is $\int_{a}^{\infty} f(x)\,dx$

Exercise:

Use the Comparison Theorem to determine whether the integral is convergent or divergent.

- $\int_{1}^{\infty} \frac{1+\sin^2 x}{\sqrt{x}}\,dx$